AD 697 713

ACOUSTIC WAVES IN THE IONOSPHERE

D. P. Hoult

Pennsylvania State University
University Park, Pennsylvania

1 October 1969

DISCLAIMER NOTICE

THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

THE PENNSYLVANIA STATE UNIVERSITY

IONOSPHERIC RESEARCH

Scientific Report No. 339

ACOUSTIC WAVES IN THE IONOSPHERE

by

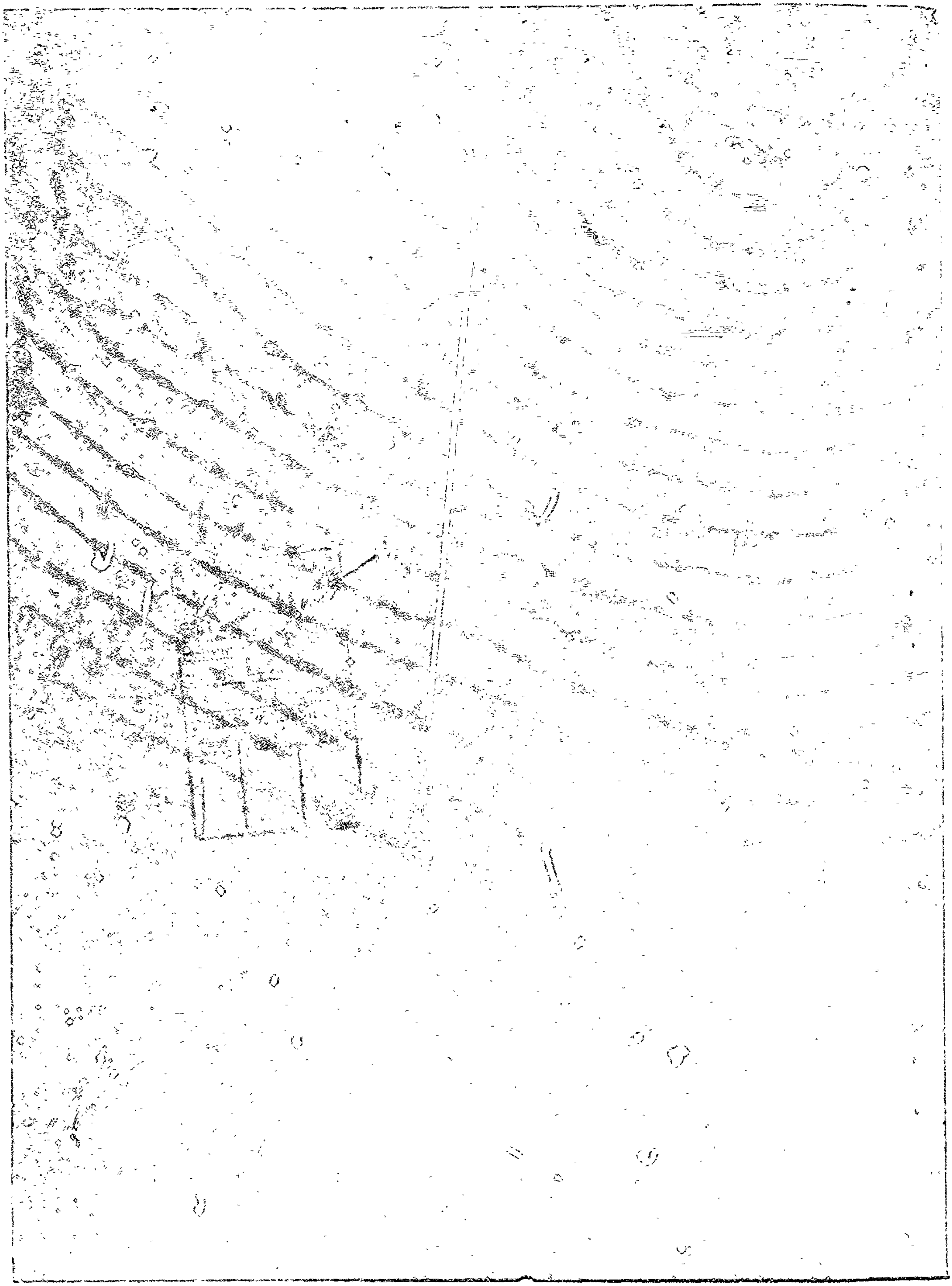
D. E. Hoult

October 1, 1969

The research reported in this document has been sponsored by the U.S. Army Ballistics Research Laboratory, Aberdeen Proving Ground, Maryland, under Contract DAHC[illegible]-0031 and, in part, by the National Science Foundation under Grant NSF GP-5811.

IONOSPHERE RESEARCH LABORATORY

University Park, Pennsylvania Contract DAHC[illegible]



Ionospheric Research

Contract DAHC04-68-C-0034

Scientific Report No. 339

on

"Acoustic Waves in the Ionosphere"

by

D. P. Hoult

October 1, 1969

"The research reported in this document has been sponsored by the U.S. Army Ballistics Research Laboratory, Aberdeen Proving Ground, Maryland, under Contract DAHC04-68-0034 and, in part, by the National Science Foundation under Grant NSF GP-5611."

Submitted by: L. C. Hale (aw)

L. C. Hale, Associate Professor of Electrical Engineering

Approved by: A. H. Waynick

A. H. Waynick, Director, Ionosphere Research Laboratory

Ionosphere Research Laboratory
The Pennsylvania State University
University Park, Pennsylvania 16802

TABLE OF CONTENTS

ABSTRACT i

I. Introduction 1

II. METHODOLOGY AND DATA COLLECTION. 3

III. THEORETICAL MODEL 5

IV. THE POWER SPECTRUM. 8

V. DISCUSSION. 11

VI. References 13

ABSTRACT

Acoustic wave characteristics in the upper atmosphere are deduced from temporal and spatial electron density variations obtained with the Arecibo back-scatter sounder. A theory is developed which relates the observed wave characteristics to the power spectrum of the wave sources.

ACOUSTIC WAVES IN THE IONOSPHERE

I. Introduction

Observations[1] of the lower ionosphere and upper D region (the region from 80 km to 150 km) show that there the atmosphere is in a state of random motion. Since only a limited amount of statistical information can be obtained from rocket borne experiments, it is natural to attempt to use ground based measurements to obtain information about the random state of the lower ionosphere. This work is a report of one attempt to obtain a reasonably complete physical picture of one class of ionospheric motions: those corresponding to random internal acoustic waves.

The radar facility at Arecibo, Puerto Rico, offers the best available spatial and temporal resolution of the backscattered power, and the least ambiguity as to the cross-section of the scattering process. For altitudes greater than 110 km, the mean free path of the gas is greater than the wave length of the radio wave (70 cm), which is in turn greater than the Debye length of the gas (10 cm). Under these circumstances, it can by shown that Thompson scattering occurs, so that the backscattered power is proportional to the electron density of the gas.[2] Furthermore, the conditions of the theory are approximately met down to about 80 km; electron densities determined by rocket borne measurements are in fairly good agreement with the radar data.[3] Thus, a measure of electron density is available in the 80-150 km region, with a spatial resolution of about 6 km in the horizontal and vertical, a time resolution of 10^{-1} sec. and a 10 percent accuracy in backscattered power.

We shall be dealing then with a random electron density which depends on spatial position and time. However, a further simplification is available in the 80-150 km region, when data are taken only around noon. It is thus: if the phenomena which we wish to observe have time scales τ less than about 30 minutes, we may suppose that the electron density variations are proportional to the bulk fluid density. For example, to have diffusion be unimportant in a cube of size l during a time τ, we must have $\tau < \frac{l^2}{D}$. But a typical diffusion coefficient might be 10^6 cm^2/sec at 110 km, if l is 6 km, τ must be less than several days. Again, if photo chemical reactions are present, and have a characteristic reaction time τ_R, then $\tau_R > \tau$. At around 100 km, τ_R might be one second. Thus, one would expect the electron density fluctuations to be in photochemical equilibrium. When the sun is overhead, this equilibrium implies a constant electron concentration. Finally, it has been shown that electron density fluctuations are the same as the neutral density fluctuation in an acoustic wave in this altitude region(4), even though the electron velocity is not the same as that of the bulk fluid, due to magneto-ionic effects.

For the purpose of the following discussion, we assume that the backscattered power received by the Arecibo antenna measures the bulk density of the gas. The purpose of this paper is to describe how the statistics of the bulk density fluctuations can be used to determine the motions of the gas. In particular, we show how such measurements as described above can be used to deduce the source strength of random acoustic waves. The methodology of the analysis required to distinguish random waves from other kinds of random motions, such as turbulence, is reviewed. The data from the present observations is presented. A theory is presented which is consistent with the present observations.

Using this theory, the power spectrum of the sources of the acoustic waves is determined. A discussion of the results, and their geophysical significance, is given.

II. Methodology and Data Collection

The correlation method has been described previously(5), (6) and will be presented in an abbreviated form here. Suppose the signal received, contains both a component due to random waves, and a component due to noise, n, which may include receiver noise, or turbulence. The crux of the idea is that the component due to waves must be correlated over larger distances and times than the noise if the waves are physically distinguishable from the noise.

Suppose z is vertical upward, and the waves satisfy a dispersion relation

$$\omega = \omega(\vec{k}) \tag{1}$$

where $\vec{k}$ is the wave number vector. Then we assume the signal may be written as if ξ is the percent fluctuation in electron density.

$$\xi = \int dA\,(z, \vec{k})\, e^{i\omega t}\, e^{i\vec{k}\,\vec{x}} + n(\vec{x}, t) \tag{2}$$

Here, A is the random amplitude of the wave field. We suppose that the waves are not altered by the presence of the noise, so that A and n are uncorrelated. Then the correlation function has the form

$$\begin{aligned} R &= \{\xi(\vec{x}, t)\,\xi(\vec{x} + \Delta\vec{x}, t + \Delta\vec{t})\} \\ &= \int P(z, \Delta z, \vec{k})\, d\vec{k}\; e^{i\omega\Delta t}\, e^{-i\vec{k}\Delta x} + \rho(\Delta\vec{x}, z, \Delta t) \end{aligned} \tag{3}$$

Here ρ is the correlation function of the noise. The correlation functions depend on z as well as Δz due to the inhomogenity of the media.

Now suppose this data ξ is collected while the antenna is pointed in a fixed direction $\vec{s}$. Then $\Delta\vec{x} = \Delta\vec{s}$; that is, the optical stages

in (3) are taken along $\vec{s}$. If Δt and $\Delta\vec{x}$ are allowed to become very large, then ρ will vanish exponentially rapidly[5], whereas the wave-like component of R varies algebraicly

$$\lim_{\left.\begin{matrix}\Delta t\\ \Delta\vec{x}\end{matrix}\right\}\to\infty} R = \lim_{\left.\begin{matrix}\Delta t\\ \Delta\vec{x}\end{matrix}\right\}\to\infty} \left\{ \int P(z, \Delta z, \vec{k})\ dk\ e^{i\omega\Delta t - i\vec{k}\Delta\vec{x}} \right\}$$

This integral may be approximated by the method of stationary phase, the stationary point is

$$\operatorname{grad}_{\vec{k}} \omega = \frac{\Delta\vec{x}}{\Delta t} = \frac{\Delta\vec{s}}{\Delta t}, \qquad \operatorname{grad}_{\vec{k}} (\quad) = \left(\frac{\partial}{\partial k_i}\right) \tag{4}$$

Provided $\Delta\vec{x}$, Δt are sufficiently large, all the contributions to R will come from wave numbers and frequencies which satisfy (4). It is only when Δx and Δt become large that the wave-like component of R may be separated from ρ. Hence, we obtain the following rule: <u>If the antenna is held stationary, the waves seen will be those whose group velocity is parallel to the orientation of the antenna, $\vec{s}$.</u>

Consider now a still atmosphere. The internal waves which may propagate in such an atmosphere have the following general properties (see ref. (7)): For $\vec{s}$ oriented at an angle θ to the vertical, there are two internal gravity waves $\vec{k}_1$, $\vec{k}_2$, and one acoustic wave $\vec{k}_3$ which satisfy equation 4. Thus, one is faced with determining three amplitudes from R. If winds of unknown magnitude are assumed to be present, then the difficulties are magnified.

However, there is an important simplification available when the antenna points directly vertical. In this case, there is only one solution to equation (4), which is an acoustic wave propagating vertically.

This is the case studied here.

The data were collected in December 1965 and have been reported in ref. (6). The two main features of the data which we wish to describe are shown in Fig. 1, which shows the oscillatory character of the correlation function for large Δz, Δt and Fig. 2. Fig. 2 shows the zeroes of the correlation function for large lags. The error bars are estimated as in ref. (6).

The main features of the data are:(a) the amplitude of the oscillations does not increase with Δz, and (b) the zeroes of R are nearly straight lines parallel to the Δz axis.

III. Theoretical Model

In this section we present a simple model of vertically propagating disturbance which has the following properties. 1) the dispersion relationship yields the distribution of zeroes shown in Fig. 2, and 2) the amplitude is nearly constant with altitude as in Fig. 1.

Recall that in a constant temperature atmosphere, an internal wave would grow exponentially with altitude, provided viscosity were not important. It is known[8] that for long gravity waves, viscosity is an important filter in the 100 km range. However, such waves propagate nearly horizontally, and hence, at a given altitude, viscous forces have a long time to act on a wave packet. Because viscous forces at a given altitude act only for a short time on an acoustic wave packet propagating vertically, it seems plausible that viscosity would become important only at altitudes higher than 100 km.

If c is the speed of sound, ρ is fluid density, and μ the viscosity of the gas, and H a scale height, then for acoustic waves, an appropriate

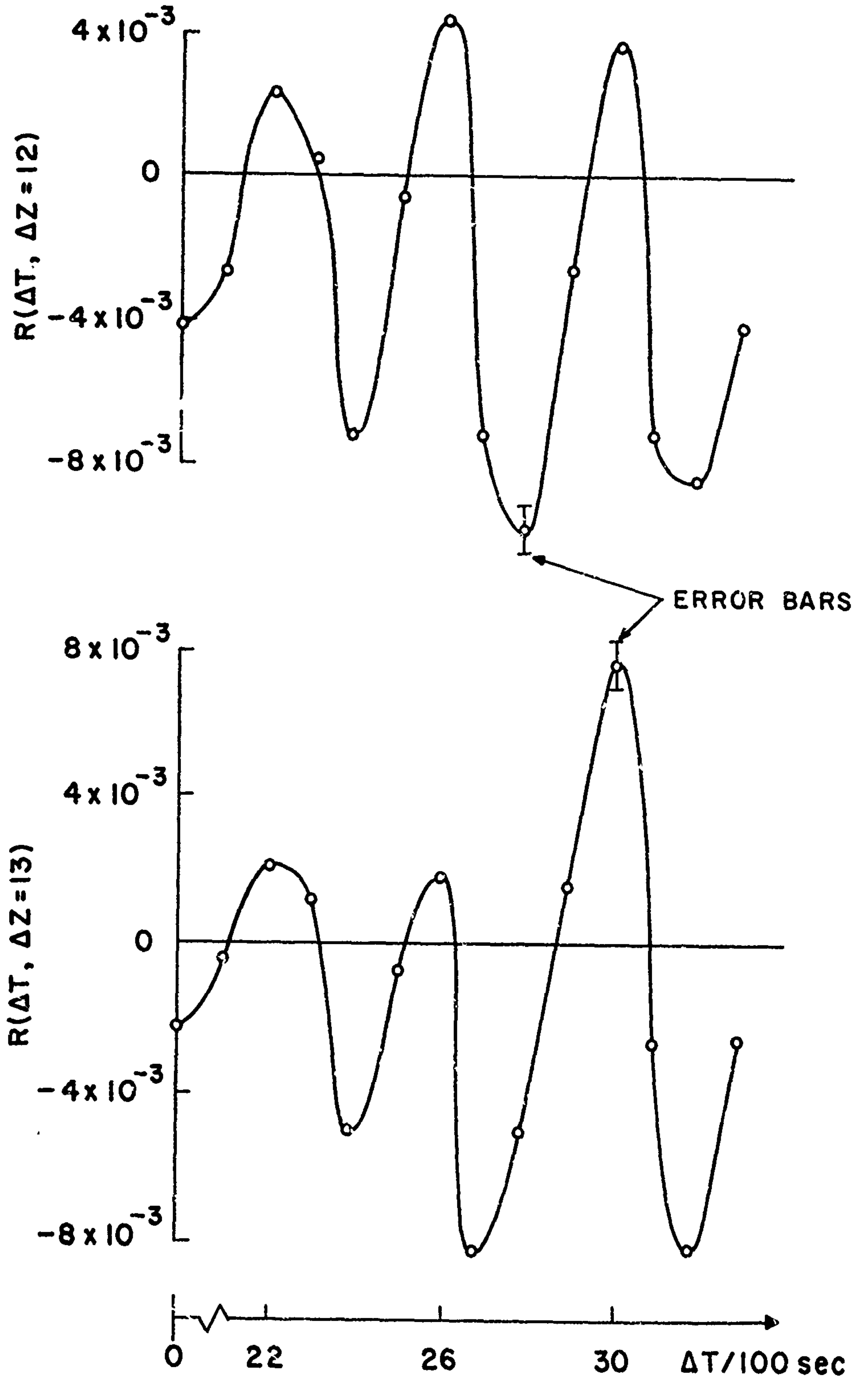


FIG. 1. BEHAVIOR OF R FOR LARGE ΔZ, ΔT

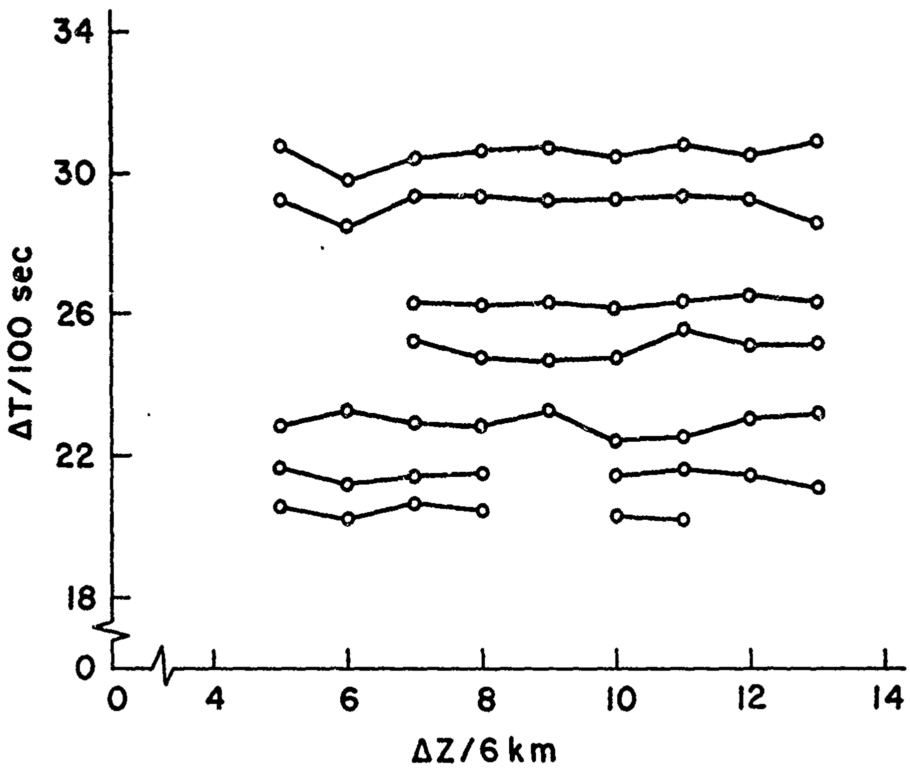


FIG. 2. —o— ZEROS OF R FOR LARGE ΔZ, ΔT

Reynolds number (the ratio of viscous to inertial forces) is

$$Re = \frac{\rho c H}{\mu} .$$

Fig. 3 shows that this parameter is quite large in the 100-200 km altitude range, indicating that viscous effects are unimportant in this region. This result is consistent with the results of Maeda (1964) who found that acoustic waves were damped by viscous action around 300 km for waves of 100 sec period (see Fig. 11 of Maeda 1964). Since the period of the oscillations in R is nearer 300 sec, Maeda's calculation suggests that, these disturbances should not be attenuated by viscosity below about 400 km. Furthermore, the detailed calculations of Yanowitch (1967) on the viscous damping of the acoustic wave may be used to compute the expected form of the correlation coefficient R. When this is done, one finds: 1) the damping effect switches on exponentially fast at some altitude, and 2) above that altitude, the oscillations in R increase in period. Neither of these effects is observed.

Fig. 5 shows that in the 80-150 km region, there is a strong temperature gradient. We now show that this temperature gradient can be invoked to explain the amplitude dependence of the data. The analysis below is analogous to that given by Maeda (1964), except that low frequency waves are considered. If D/Dt is the substantial derivative, p the pressure and u the fluid velocity taken vertical, then the equations of motion are

Continuity

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \frac{\partial u}{\partial t} = 0 \qquad (1)$$

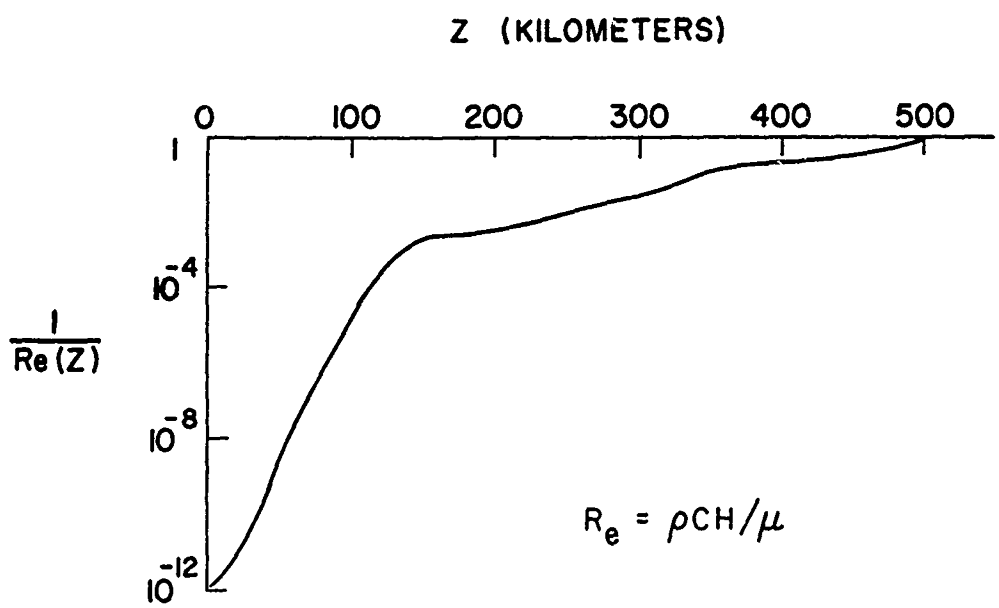


FIG. 3. REYNOLDS NUMBER VARIATION IN THE ATMOSPHERE

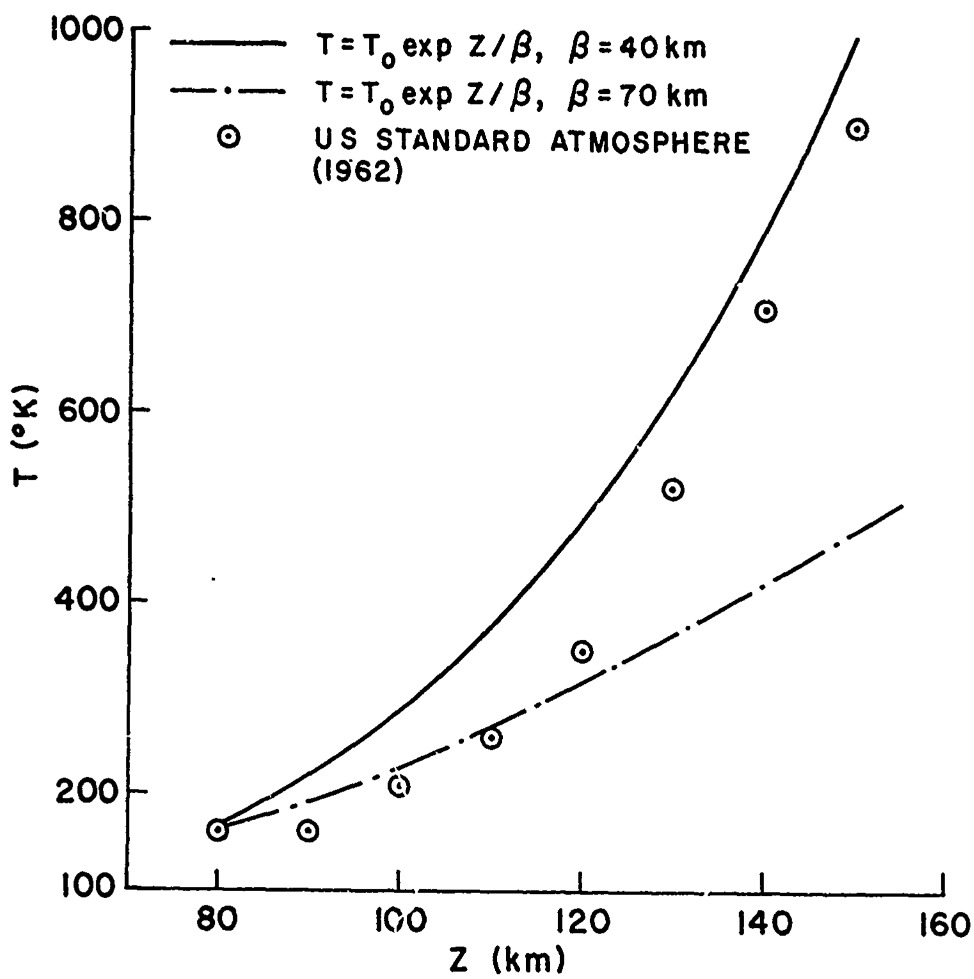


FIG. 4. TEMPERATURE AS A FUNCTION OF ALTITUDE

Momentum

$$\rho \frac{Du}{Dt} + \frac{\partial p}{\partial z} + g = 0 \tag{2}$$

Energy

$$\frac{Dp}{Dt} = c^2 \frac{D\rho}{Dt} \tag{3}$$

By linearizing these equations and cross differentiating, one obtains (see Maeda 1964)

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial z^2} + \gamma g \frac{\partial u}{\partial z} = 0 \tag{4}$$

Here $c = \sqrt{\gamma R T_o}$, the speed of sound, g is the acceleration due to gravity and γ is the ratio of specific heats.

A simple approximate solution, valid for small temperature gradients, will suffice. Set

$$u = A \left[\exp \left(\frac{g}{2} \int_o^z \frac{dz}{RT_o} \right) \right] e^{i\omega t} \hat{u}(z) \tag{5}$$

Then, upon substitution into equation (4) gives

$$\frac{d^2 \hat{u}}{dz^2} + \hat{u} \left(\frac{\omega^2}{\gamma R T_o} - \frac{g^2}{4(RT_o)^2} - \frac{g}{2RT_o} \left(\frac{\partial}{\partial z} (\ln T_o) \right) \right) \tag{6}$$

We assume that the temperature increases exponentially with altitude (see Fig. 4); hence

$$T_o = T\,(z = 80 \text{ km})\ \exp z/\beta$$

Using this, the amplitude factor in equation (5),

$$A \exp \left(\frac{-g\beta}{2RT(0)} \exp - z/\beta \right) , \tag{7}$$

varies by at most 50%, as $1 < z/\beta < 2$ in the 120-150 km altitude range. Here $T(0) = T_o(z = 80 \text{ km})$. In this range, $\hat{u}$ has the approximate form

$$\hat{u} \sim e^{ikz}, \quad k = \sqrt{\frac{\omega^2}{\gamma RT_o} - \frac{1}{4H^2} - \frac{1}{2H\beta}} \tag{8}$$

Here, H, the scale height, is RT_o/g. As $\beta \approx 40$ km, and $H \approx 8$ km, it is seen that the main effect of the temperature gradient is to reduce the exponential increase in amplitude of the wave while leaving the dispersion relationship nearly unchanged.

IV. The Power Spectrum

In this section we use the approximate solution of the preceding section to estimate the form of the correlation functions of the waves. According to equation (7), (8), the disturbance in density (which is proportional to $\hat{u}$ due to linearity) has the form

$$\xi = \int_{-\infty}^{\infty} dA\,(\omega, z) \exp \left(\frac{-g\beta}{2RT(0)} \exp - \frac{z}{\beta} \right) e^{i\omega t} e^{-ikz} + \rho$$

where k is given by equation (8). Forming the correlation coefficient R, we have

$$R = \int_{-\infty}^{\infty} < dA\,(\omega, z)\; dA^*\,(\omega, z + \Delta z) > e^{i\omega\Delta t}\, e^{-ik\Delta z}$$

$$\exp \frac{-g\beta}{2RT_o} \left[\exp - \left(\frac{z}{\beta}\right) + \exp - \left(\frac{z + \Delta z}{\beta}\right) \right]$$

In the limit of large Δt, Δz this integral has a stationary point at

$$\frac{dk}{d\omega} = \frac{\Delta t}{\Delta z}$$

which can be shown to be

$$\frac{\Delta t}{\Delta z} = \frac{\omega}{\gamma R T_o k}$$

In terms of frequencies this result may be written as

$$\omega = \sqrt{\frac{\gamma R T_o}{4H^2}\left(1 - \frac{2H}{\beta}\right)\left[\frac{\gamma R T_o (\Delta t)^2}{\gamma R T_o (\Delta t)^2 - (\Delta z)^2}\right]}$$

Referring now to figure 3, it is clear that $(\gamma R T_o)(\Delta t)^2 >> (\Delta z)^2$, so that we may expand the formula for ω as

$$\omega = \sqrt{\frac{\gamma R T_o}{4H^2}\left(1 - \frac{2H}{\beta}\right)}\left(1 + \frac{1}{2}\,\frac{(\Delta z)^2}{\gamma R T_o (\Delta t)^2}\right) \qquad (9)$$

According to this model, the acoustic cutoff frequency in a temperature gradient is

$$\omega_A = \sqrt{\frac{\gamma R T_o}{4H^2}\quad 1 - \frac{2H}{\beta}} \qquad (10)$$

which is lower by a factor $\sqrt{(1 - \frac{2H}{\beta})}$ from the isothermal case. Using formula (9) one can associate oscillations occurring in R at given values of Δz, Δt with a frequency of the source of the disturbance, ω.

The amplitude of the oscillations is easily worked out to be

$$R = P(\omega,\ z,\ \Delta z)\ A^1(z)\ \cos\left\{ \frac{(\Delta t)^2\,\gamma R T_o - (\Delta z)^2}{4H^2}\left[(1 - \frac{2H}{\beta})\right]^{1/2} + \frac{1}{4}\pi \right\}$$

where $A^1(z)$ has the form

$$\exp\left(\frac{-g\,\beta}{2RT_o}\quad\left[\exp\ -\ \frac{z}{\beta}\right]\right)$$

For $(\Delta t)^2\ \gamma RT_o >> \Delta z$, the amplitude of the oscillations may be approximated by

$$R = A(z)\ \ P(\omega,\ z,\ \Delta z)\ \ \cos\left\{\frac{\Delta t}{2H}\sqrt{\gamma RT_o\,(1 - \frac{2H}{\beta})}\ +\ \frac{1}{4}\ \pi\right\} \qquad (11)$$

It will be seen shortly that the observed oscillations are very close to the acoustic cutoff frequency. In this case, the lines of constant phase are straight lines, parallel to the Δz axis . The period of these oscillations, taking account of the finite temperature gradient, is about 310 sec at 100 km. The observed zeroes of R (lines of constant phase) (Fig. 2) form straight lines parallel to the Δz axis, and their mean spacing yields an observed period of 320 sec.

To construct a power spectrum of the source strength of the waves, a value of Δz, Δt is selected, and the amplitude of this is estimated from the data. This amplitude is a measure of the power spectrum $P(\omega,\ z,\ \Delta z)$. The frequency is found by equation (10).

Using this method, the results shown in Fig. 6 have been deduced. It is reassuring to note that the amplitudes for various Δz, Δt, reduce to a single curve when plotted this way.

V. Discussion

One might compare these results with a simple nonpropagating disturbance, of the form $u \sim e^{i\omega At}$. Experimentally, the correlation function, R, should behave as $e^{i\omega_A \Delta t}$ independent of z. Actually, in the region of Δz, Δt space where the oscillations are observed, this is nearly the case. Clearly, the equations which would describe small oscillations and the acoustic cutoff frequency are the same as those presented in Section III. For all practical purposes, then, these waves are very nearly standing waves.

Because of this fact, a calculation of the energy flux carried by the wave is uncertain, involving the difference of two small numbers. The present estimate is 1.4×10^{-4} watts/m^2. This value is comparable with that predicted by Hines (1965) for the energy flux due to internal gravity waves. Although it can be shown that the disturbance from a point source of energy in the lower atmosphere, ignoring the ground effect, is equally divided between acoustic and gravity waves, it does not necessarily follow that the energy flux at 100 km from these two wave modes is equal. The present data do suggest this.

Another explanation of the data supposes that the disturbances observed have their origin in internal gravity waves at a ground level. Consider an internal wave very near the gravity wave cutoff frequency at ground level, propagating upward. As such a wave encounters the temperature variations in the atmosphere, it will, in some regions,

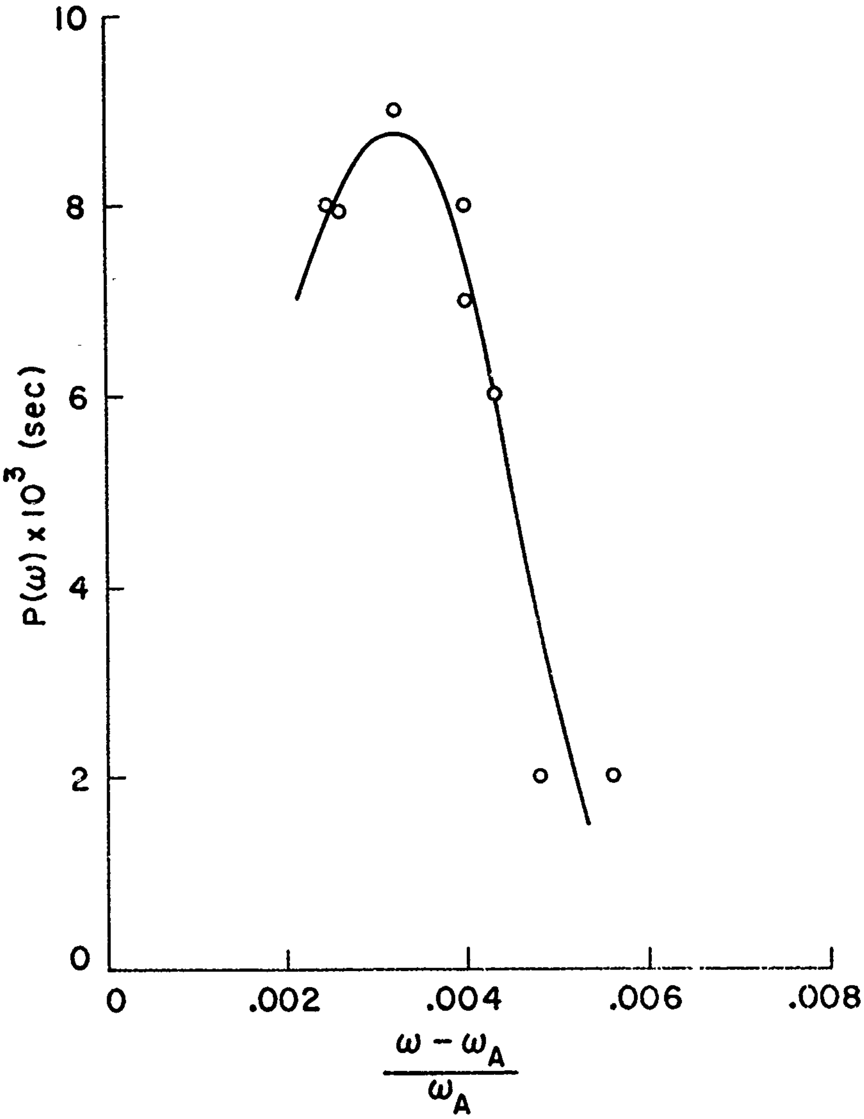


FIG. 5. THE POWER SPECTRUM OF THE ACOUSTIC WAVE SOURCES

become damped. We suppose that as the wave propagates through the temperature field of the atmosphere, a mode conversion process occurs, in which the damped gravity wave becomes an acoustic wave when the temperature drops so far as to allow the disturbance to propagate as an acoustic wave.

According to this model, the power spectrum of the waves should be bounded above by ω_G, the gravity wave cutoff frequency at ground level, and below by ω_A, the acoustic cutoff frequency at 100 km. A calculation on these lines shows that one can adequately explain the width of the spectrum observed with these ideas. However, one would expect that the energy flux would be an order of magnitude down from that of internal gravity waves with this model. It is not clear if this is the case.

Finally, it should be emphasized that the methods presented here may be used to analyze internal gravity waves also; provided the antenna is directed at an angle to the vertical.

VI. References

(1) Johnson, F. S. and Wilkins, E. M. 1965. JGR 70. Pp 1281-1284, P. 4063.

(2) Gordon, W. E. (1958) Proc. IRE, 46, 1824.

(3) Brace, L. H., Carlson, H. C., and Mahajan, K. K. 1969. JGR 74. P. 1883.

(4) Pierce, A. and Kantor, G. 1968. J. Atmos. and Terr. Phys. 30. P.. 1497.

(5) Hoult, D. P. 1966. Phys. Fluids 9. 1565.

(6) Hoult, D. P. 1966. Space Res. VII. 1059.

(7) Hines, C. O. 1960. Can. J. Phys. 1441.

(8) Hines, C. O. 1965. JGR 70. P.. 177.

(9) Maeda 1964. JGR 69. P. 1381.

(10) Yanowitch 1967. Can. J. Phys. 45. 2003